

Student Name \_\_\_\_\_  
 Teacher Name \_\_\_\_\_  
 School \_\_\_\_\_

The College Board  
 Advanced Placement Examination  
 PHYSICS C  
 SECTION II

TABLE OF INFORMATION FOR 1999

CONSTANTS AND CONVERSION FACTORS		UNITS		PREFIXES																																		
		Name	Symbol	Factor	Prefix	Symbol																																
1 unified atomic mass unit,	$1u = 1.66 \times 10^{-27} \text{ kg}$ $= 931 \text{ MeV}/c^2$	meter	m	$10^9$	giga	G																																
Proton mass,	$m_p = 1.67 \times 10^{-27} \text{ kg}$	kilogram	kg	$10^6$	mega	M																																
Neutron mass,	$m_n = 1.67 \times 10^{-27} \text{ kg}$	second	s	$10^3$	kilo	k																																
Electron mass,	$m_e = 9.11 \times 10^{-31} \text{ kg}$	ampere	A	$10^{-2}$	centi	c																																
Magnitude of the electron charge,	$e = 1.60 \times 10^{-19} \text{ C}$	kelvin	K	$10^{-3}$	milli	m																																
Avogadro's number,	$N_0 = 6.02 \times 10^{23} \text{ mol}^{-1}$	mole	mol	$10^{-6}$	micro	$\mu$																																
Universal gas constant,	$R = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$	hertz	Hz	$10^{-9}$	nano	n																																
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23} \text{ J}/\text{K}$	newton	N	$10^{-12}$	pico	p																																
Speed of light,	$c = 3.00 \times 10^8 \text{ m}/\text{s}$	pascal	Pa	VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th><math>\theta</math></th> <th>Sin <math>\theta</math></th> <th>Cos <math>\theta</math></th> <th>Tan <math>\theta</math></th> </tr> </thead> <tbody> <tr> <td><math>0^\circ</math></td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td><math>30^\circ</math></td> <td>1/2</td> <td><math>\sqrt{3}/2</math></td> <td><math>\sqrt{3}/3</math></td> </tr> <tr> <td><math>37^\circ</math></td> <td>3/5</td> <td>4/5</td> <td>3/4</td> </tr> <tr> <td><math>45^\circ</math></td> <td><math>\sqrt{2}/2</math></td> <td><math>\sqrt{2}/2</math></td> <td>1</td> </tr> <tr> <td><math>53^\circ</math></td> <td>4/5</td> <td>3/5</td> <td>4/3</td> </tr> <tr> <td><math>60^\circ</math></td> <td><math>\sqrt{3}/2</math></td> <td>1/2</td> <td><math>\sqrt{3}</math></td> </tr> <tr> <td><math>90^\circ</math></td> <td>1</td> <td>0</td> <td><math>\infty</math></td> </tr> </tbody> </table>			$\theta$	Sin $\theta$	Cos $\theta$	Tan $\theta$	$0^\circ$	0	1	0	$30^\circ$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$	$37^\circ$	3/5	4/5	3/4	$45^\circ$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$53^\circ$	4/5	3/5	4/3	$60^\circ$	$\sqrt{3}/2$	1/2	$\sqrt{3}$	$90^\circ$	1	0	$\infty$
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$90^\circ$	1	0	$\infty$																																			
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ $= 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$	joule	J																																			
	$hc = 1.99 \times 10^{-25} \text{ J} \cdot \text{m}$ $= 1.24 \times 10^3 \text{ eV} \cdot \text{nm}$	watt	W																																			
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$	coulomb	C																																			
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$	volt	V																																			
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} (\text{T} \cdot \text{m})/\text{A}$	ohm	$\Omega$																																			
Magnetic constant,	$k^1 = \mu_0/4\pi = 10^{-7} (\text{T} \cdot \text{m})/\text{A}$	henry	H																																			
Universal gravitational constant,	$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$	farad	F																																			
Acceleration due to gravity at the Earth's surface,	$g = 9.8 \text{ m}/\text{s}^2$	tesla	T																																			
1 atmosphere pressure,	1 atm = $1.0 \times 10^5 \text{ N}/\text{m}^2$ $= 1.0 \times 10^5 \text{ Pa}$	degree Celsius	$^\circ\text{C}$																																			
1 electron volt,	1 eV = $1.60 \times 10^{-19} \text{ J}$	electron-volt	eV																																			
1 angstrom,	1 $\text{\AA} = 1 \times 10^{-10} \text{ m}$																																					

The following conventions are used in this examination.

- I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.
- II. The direction of any electric current is the direction of flow of positive charge (conventional current).
- III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

This insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

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## MECHANICS

$$v = v_0 + at$$

$$s = s_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$\Sigma \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F} dt = \Delta\mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \leq \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{s}$$

$$K = \frac{1}{2}mv^2$$

$$P = \frac{dW}{dt}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\Sigma \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \Sigma mr^2$$

$$\mathbf{r}_{cm} = \Sigma m\mathbf{r} / \Sigma m$$

$$v = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$F_G = -\frac{Gm_1m_2}{r^2}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

$a$  = acceleration  
 $F$  = force  
 $f$  = frequency  
 $h$  = height  
 $I$  = rotational inertia  
 $J$  = impulse  
 $K$  = kinetic energy  
 $k$  = spring constant  
 $\ell$  = length  
 $L$  = angular momentum  
 $m$  = mass  
 $N$  = normal force  
 $P$  = power  
 $p$  = momentum  
 $r$  = distance  
 $s$  = displacement  
 $T$  = period  
 $t$  = time  
 $U$  = potential energy  
 $v$  = velocity or speed  
 $W$  = work  
 $x$  = displacement  
 $\mu$  = coefficient of friction  
 $\theta$  = angle  
 $\boldsymbol{\tau}$  = torque  
 $\omega$  = angular speed  
 $\alpha$  = angular acceleration

## ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q}{r}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$\mathbf{F} = \int I d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\Phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

$A$  = area  
 $B$  = magnetic field  
 $C$  = capacitance  
 $d$  = distance  
 $E$  = electric field  
 $\mathcal{E}$  = emf  
 $F$  = force  
 $I$  = current  
 $L$  = inductance  
 $\ell$  = length  
 $n$  = number of loops of wire  
           per unit length  
 $P$  = power  
 $Q$  = charge  
 $q$  = point charge  
 $R$  = resistance  
 $r$  = distance  
 $t$  = time  
 $U$  = potential or stored energy  
 $V$  = electric potential  
 $v$  = velocity or speed  
 $\rho$  = resistivity  
 $\Phi_m$  = magnetic flux  
 $\kappa$  = dielectric constant

**GEOMETRY AND TRIGONOMETRY**

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

Parallelepiped

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right Triangle

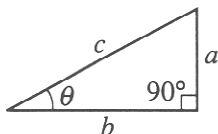
$$a^2 + b^2 = c^2$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

$A$  = area  
 $C$  = circumference  
 $V$  = volume  
 $S$  = surface area  
 $b$  = base  
 $h$  = height  
 $\ell$  = length  
 $w$  = width  
 $r$  = radius



**CALCULUS**

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1$$

$$\int e^x dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x dx = \sin x$$

$$\int \sin x dx = -\cos x$$

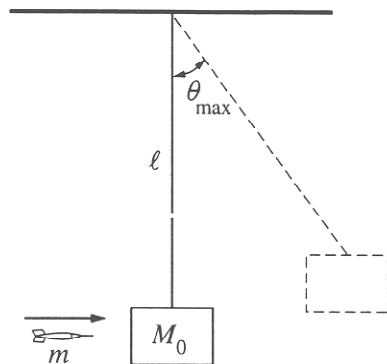
## PHYSICS C

## SECTION II, MECHANICS

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech 1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass  $m$ , is fired with the gun very close to a wooden block of mass  $M_0$ , which hangs from a cord of length  $\ell$  and negligible mass, as shown above. Assume the size of the block is negligible compared to  $\ell$ , and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle  $\theta_{\max}$  from the vertical. Express your answers to the following in terms of  $m$ ,  $M_0$ ,  $\ell$ ,  $\theta_{\max}$ , and  $g$ .

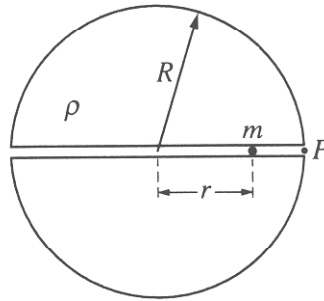
- Determine the speed  $v_0$  of the dart immediately before it strikes the block.
- The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.
- At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses
Protractor	5 m of string	Five more blocks of mass $M_0$
Spring		

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.

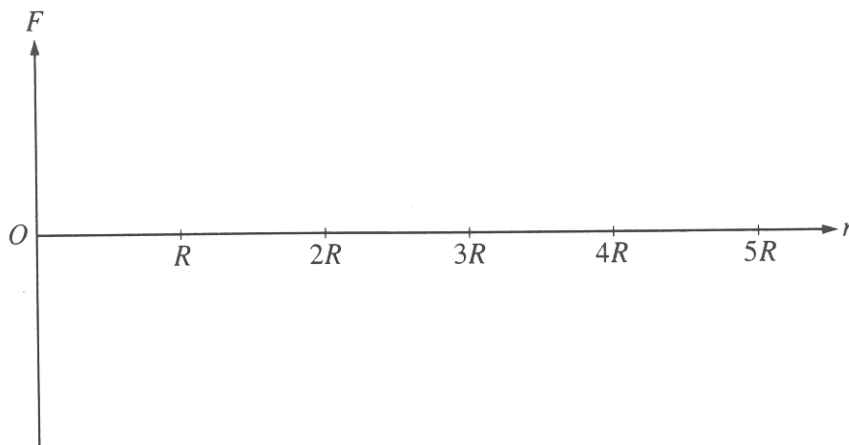
**GO ON TO THE NEXT PAGE**

- (d) The dart is now shot into a block of wood that is fixed in place. The block exerts a force  $\mathbf{F}$  on the dart that is proportional to the dart's velocity  $\mathbf{v}$  and in the opposite direction, that is  $\mathbf{F} = -b\mathbf{v}$ , where  $b$  is a constant. Derive an expression for the distance  $L$  that the dart penetrates into the block, in terms of  $m$ ,  $v_0$ , and  $b$ .



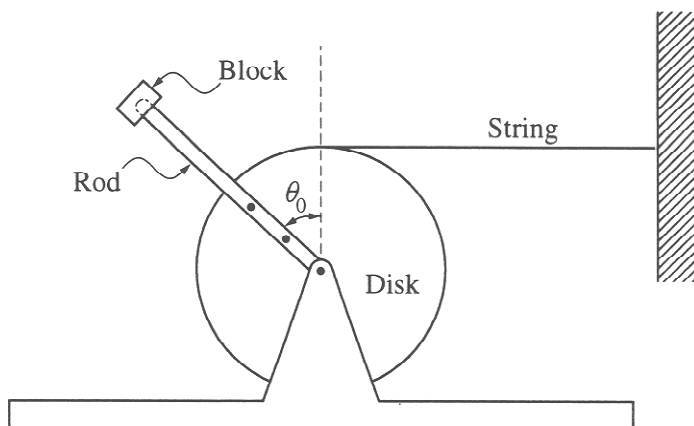
Mech 2. A spherical, nonrotating planet has a radius  $R$  and a uniform density  $\rho$  throughout its volume. Suppose a narrow tunnel were drilled through the planet along one of its diameters, as shown in the figure above, in which a small ball of mass  $m$  could move freely under the influence of gravity. Let  $r$  be the distance of the ball from the center of the planet.

- (a) Show that the magnitude of the force on the ball at a distance  $r < R$  from the center of the planet is given by  $F = -Cr$ , where  $C = \frac{4}{3} \pi G \rho m$ .
- (b) On the axes below, sketch the force  $F$  on the ball as a function of distance  $r$  from the center of the planet.



The ball is dropped into the tunnel from rest at point  $P$  at the planet's surface.

- (c) Determine the work done by gravity as the ball moves from the surface to the center of the planet.
- (d) Determine the speed of the ball when it reaches the center of the planet.
- (e) Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.
- (f) Write an equation that could be used to calculate the time it takes the ball to move from point  $P$  to the center of the planet. It is not necessary to solve this equation.



Mech 3. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass =  $3m$ , radius =  $R$ , moment of inertia about center  $I_D = \frac{3}{2} mR^2$

Rod: mass =  $m$ , length =  $2R$ , moment of inertia about one end  $I_R = \frac{4}{3} mR^2$

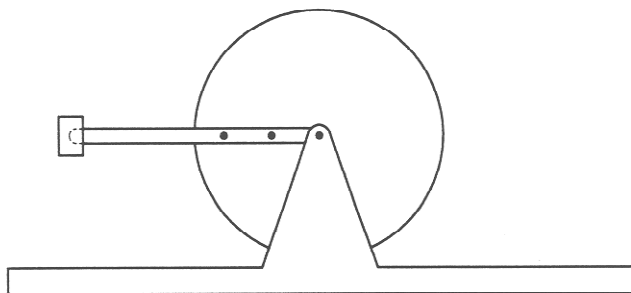
Block: mass =  $2m$

The system is held in equilibrium with the rod at an angle  $\theta_0$  to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of  $m$ ,  $R$ ,  $\theta_0$ , and  $g$ .

- (a) Determine the tension in the string.

The string is now cut, and the disk-rod-block system is free to rotate.

- (b) Determine the following for the instant immediately after the string is cut.
- i. The magnitude of the angular acceleration of the disk
  - ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

- (c) Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.

**STOP**

END OF SECTION II, MECHANICS

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, MECHANICS, ONLY. DO NOT TURN TO ANY OTHER TEST MATERIALS.

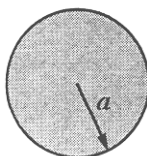
PHYSICS C

SECTION II, ELECTRICITY AND MAGNETISM

Time—45 minutes

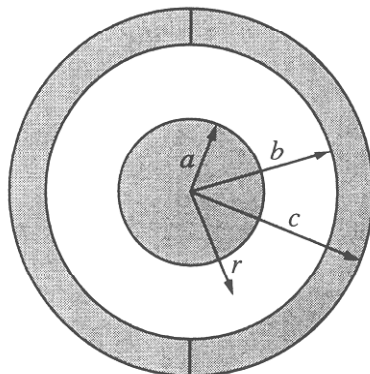
3 Questions

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E&M 1. An isolated conducting sphere of radius  $a = 0.20$  m is at a potential of  $-2,000$  V.

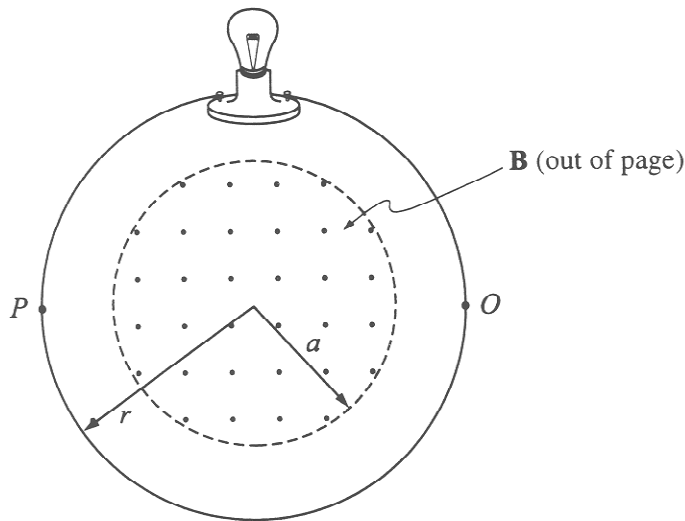
- (a) Determine the charge  $Q_0$  on the sphere.



The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius  $b = 0.40$  m and outer radius  $c = 0.50$  m, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

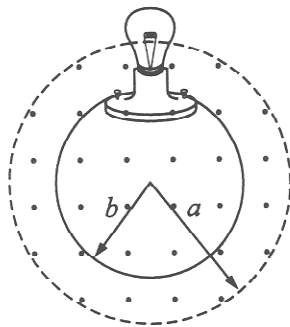
- (b) Determine the magnitude of the electric field in the following regions as a function of the distance  $r$  from the center of the inner sphere.
  - i.  $r < a$
  - ii.  $a < r < b$
  - iii.  $b < r < c$
  - iv.  $r > c$
- (c) Determine the magnitude of the potential difference between the sphere and the conducting shell.
- (d) Determine the capacitance of the spherical capacitor.





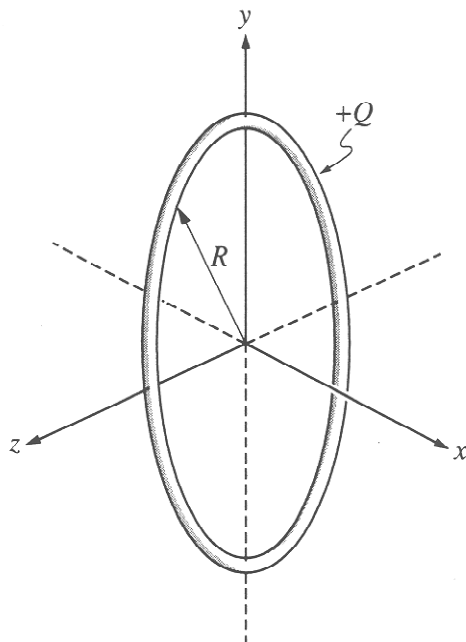
E&M 2. A uniform magnetic field  $\mathbf{B}$  exists in a region of space defined by a circle of radius  $a = 0.60\text{ m}$  as shown above. The magnetic field is perpendicular to the page and increases out of the page at a constant rate of  $0.40\text{ T/s}$ . A single circular loop of wire of negligible resistance and radius  $r = 0.90\text{ m}$  is connected to a lightbulb with a resistance  $R = 5.0\ \Omega$ , and the assembly is placed concentrically around the region of magnetic field.

- (a) Determine the emf induced in the loop.
- (b) Determine the magnitude of the current in the circuit. On the figure above, indicate the direction of the current in the loop at point  $O$ .
- (c) Determine the total energy dissipated in the lightbulb during a  $15\text{ s}$  interval.



The experiment is repeated with a loop of radius  $b = 0.40\text{ m}$  placed concentrically in the same magnetic field as before. The same lightbulb is connected to the loop, and the magnetic field again increases out of the page at a rate of  $0.40\text{ T/s}$ . Neglect any direct effects of the field on the lightbulb itself.

- (d) State whether the brightness of the bulb will be greater than, less than, or equal to the brightness of the bulb in part (a). Justify your answer.



E&M 3. The nonconducting ring of radius  $R$  shown above lies in the  $yz$ -plane and carries a uniformly distributed positive charge  $Q$ .

(a) Determine the electric potential at points along the  $x$ -axis as a function of  $x$ .

(b) i. Show that the  $x$ -component of the electric field along the  $x$ -axis is given by

$$E_x = \frac{Qx}{4\pi\epsilon_0(R^2 + x^2)^{\frac{3}{2}}}.$$

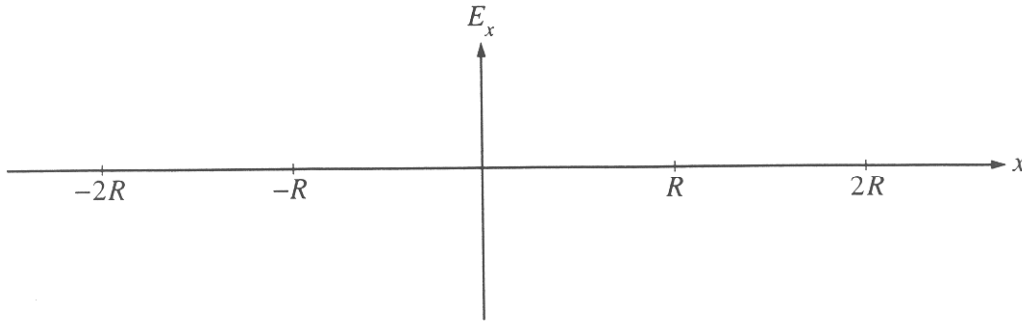
ii. What are the  $y$ - and  $z$ - components of the electric field along the  $x$ -axis?

(c) Determine the following.

i. The value of  $x$  for which  $E_x$  is a maximum

ii. The maximum electric field  $E_{x \text{ max}}$

(d) On the axes below, sketch  $E_x$  versus  $x$  for points on the  $x$ -axis from  $x = -2R$  to  $x = +2R$ .



(e) An electron is placed at  $x = R/2$  and released from rest. Qualitatively describe its subsequent motion.

**STOP**

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT TURN TO ANY OTHER TEST MATERIALS.

**NO TEST MATERIAL ON THIS PAGE**

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